Assignment 4

Due on Monday April 6^{th} (11:59 PM)

- Please read the rules for assignments on the course webpage: (`https://ece.uwaterloo.ca/~smzahedi/crs/ece752/`).
- Use Piazza or directly contact the instructor (smzahedi@uwaterloo.ca) with any questions.
- For all questions, you must show your work. Final answers alone will not receive full credit.

1. All-pay Auction (30 points). In an all-pay auction, bidders submit bids, and the object is allocated to bidder submitting the highest bid. The unusual feature is that ALL bidders pay their bids (regardless of who wins). Assume we are in the symmetric environment with independent private values.

a. (10 points) Find the unique candidate for a symmetric, increasing equilibrium.

b. (10 points) Verify that it is indeed an equilibrium for the case of $F(v) = v$ on $[0, 1]$.

c. (10 points) Is the all-pay auction "standard"? Maintain $F(v) = v$ on $[0, 1]$ and let n=2. Find the bid of a bidder with $v = 1/2$ in the symmetric equilibria or the first-price, second-price, and all-pay auctions. Explain intuitively the reason for their ranking.

2. Disclosure Game (20 points). Consider the following game. There are two agents: sender (S) and receiver (R). Sender has a type θ from set Θ, over which there is a common prior μ_0. Sender takes action by sending a message $m \in M_\theta = [0, \theta]$. Receiver observes m, updates belief about θ to μ, and selects action by responding $y \in Y$. Receiver's utility is $u_R = (m, y, \theta) = -(y - \theta)^2$ and sender's utility is $u_S(m, y, \theta) = y$.

a. (10 points) What is receiver's best response given m and μ (i.e., what is $BR_R(\mu, m)$?

b. (10 points) Prove that in any PBE, receiver perfectly infers θ from m (i.e, prove that in any PBE, sender sends a unique message for each type).

3. Modified Disclosure Game (40 points). Consider the disclosure game outlined in (2). Suppose now that the sender does not always know θ. In particular, sender knows θ only with probability $p < 1$. When the sender does not know θ, then the sender cannot report θ and can only send a null message $\emptyset$. Suppose that $\Theta = \{0, 1, 2, \ldots, 10\}$, μ_0 is uniform on Θ. Let $M_\theta = \{\emptyset, \theta\}$ be the two actions available to the sender when the sender knows θ, and $M_\emptyset = \{\emptyset\}$ be the only action available to the sender when the sender does not know θ.

a. (10 points) Show that if the sender knows $\theta = 0$, then the sender has strict incentive to send $m = \emptyset$.

b. (10 points) Suppose that there is i for which the sender weakly prefers $m = \emptyset$ to $m = i$ if the sender knows $\theta = i$. Show then for any $j < i$, the sender strictly prefers $m = \emptyset$ to $m = j$ if the sender knows $\theta = j$.

c. (10 points) Suppose we are looking for PBE in which the sender sends $m = \emptyset$ if the sender knows that $\theta = \{0, ..., k\}$ and sends $m = \theta$ if the sender knows that $\theta > k$. Calculate the sender's utility, π_k, for sending $m = \emptyset$.

d. (10 points) Show that PBE in (c) exists if and only if $k \leq \pi_k \leq k + 1$, and conclude that such PBE exists only if $k \leq 4$.