

$$1 \quad 0 = 1 + (-1)$$

$$\begin{aligned} \therefore 0 &= 0\alpha = (1 + (-1))\alpha \\ &= 1\alpha + (-1)\alpha \\ &= \alpha + (-1)\alpha \end{aligned}$$

$\therefore (-1)\alpha$ is the additive inverse of α .

$$2. \quad 0 = 1 + (-1)$$

$$\begin{aligned} \therefore \vec{0} &= 0\vec{v} = (1 + (-1))\vec{v} \\ &= 1\vec{v} + (-1)\vec{v} \\ &= \vec{v} + (-1)\vec{v} \end{aligned}$$

$\therefore (-1)\vec{v}$ is the additive inverse of $\vec{v}$.

$$3. a) \quad 3\sqrt{2} \angle -45^\circ \quad \text{or} \quad 3\sqrt{2} \angle 315^\circ$$

$$\text{or} \quad 3\sqrt{2} \angle -\frac{\pi}{4} \quad \text{or} \quad 3\sqrt{2} \angle \frac{7\pi}{4}$$

$$b) \quad 4\sqrt{2} \angle 135^\circ \quad \text{or} \quad 4\sqrt{2} \angle -225^\circ$$

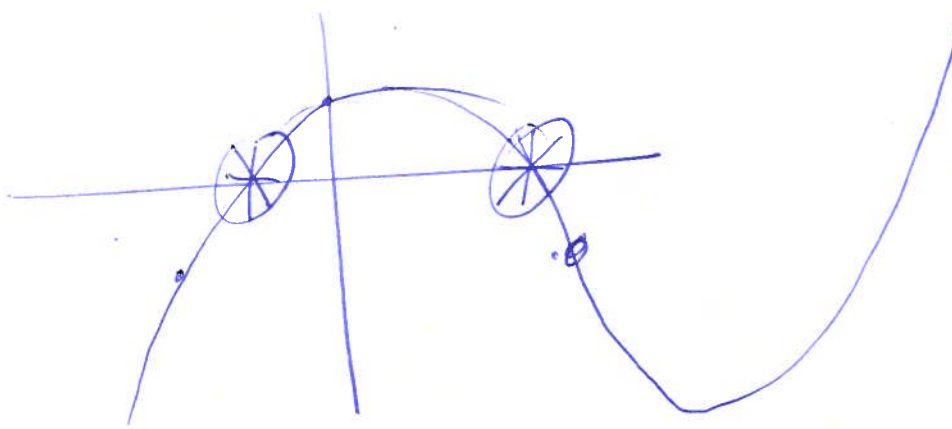
$$\text{or} \quad 4\sqrt{2} \angle \frac{3\pi}{4} \quad \text{or} \quad 4\sqrt{2} \angle \frac{7\pi}{4}$$

$$c) \quad 1 \angle \frac{\pi}{3}$$

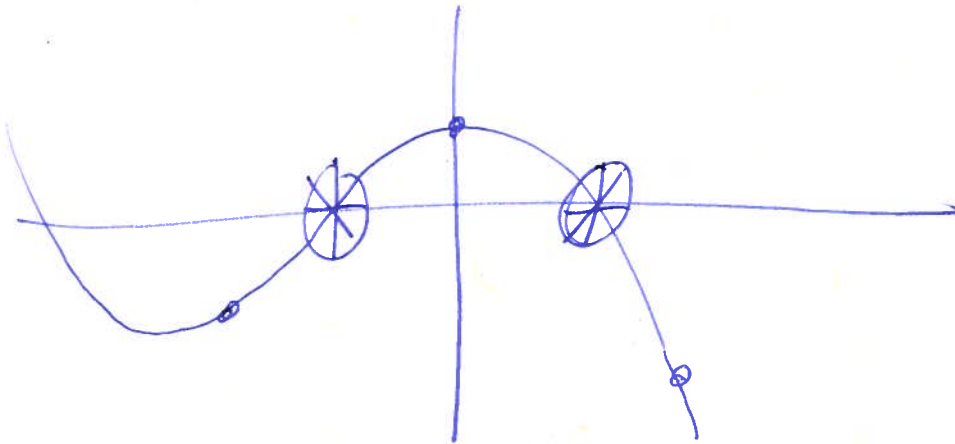
$$\begin{aligned} 4. \quad \frac{3+4j}{4-3j} &= \frac{3+4j}{4-3j} \cdot \frac{4+3j}{4+3j} = \frac{12+12j^2+9j+16j}{25} \\ &= j \end{aligned}$$

$$5 \quad -135 + 923j$$

6



or



In either case, all roots are real.
— no complex roots.

7. The first is $2(1, 1/2, 1/4, 1/8, \dots)$

The second is $(-6, 0, 0, 0, \dots) + 3(1, 1/2, 1/4, 1/8, \dots)$

$$\langle 2\vec{u}, \vec{v} + 3\vec{u} \rangle = 2\langle \vec{u}, \vec{v} \rangle + 6\langle \vec{u}, \vec{u} \rangle$$

$$= 2 \cdot (-6) + 6 \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k$$

$$= -12 + 6 \frac{1}{1 - \frac{1}{4}} = -12 + 6 \cdot \frac{4}{3}$$

$$= -12 + 8 = -4$$

Note:

if you assumed the
second was

$$-3(1, -1/2, 1/4, -1/8, +1/16, -1/32, \dots)$$

then your answer will be

$$-3 \frac{2}{1 + 1/4} = -\frac{24}{5}$$

8. $\alpha \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

Two distinct subspaces are

$\alpha \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ all multiples of $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\beta \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ all multiples of $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

Where correct answers are:

The space itself and
just the zero vector $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$.

9 $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is of the form but $2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$,

and if the first entries are 2, then the
last entry must be 4.

$\therefore \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ is not in the set.

$\therefore$ it is not a subspace.

10. $\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$ are all in
to $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, as will any linear combination
of these three.

$$\begin{aligned} 11. \quad \langle \alpha \vec{u} + \beta \vec{v}, \vec{w} \rangle &= \alpha \langle \vec{u}, \vec{w} \rangle + \beta \langle \vec{v}, \vec{w} \rangle \\ &= \alpha \langle \vec{u}, \vec{w} \rangle + \beta \langle \vec{v}, \vec{w} \rangle \\ &= \alpha \cdot 0 + \beta \cdot 0 = 0. \end{aligned}$$

$$\begin{aligned} 12. \quad \text{if } \langle \vec{u}_1, \vec{v} \rangle = 0 \text{ and } \langle \vec{u}_2, \vec{v} \rangle = 0 \\ \text{then } \langle \vec{u}_1 + \vec{u}_2, \vec{v} \rangle &= \langle \vec{u}_1, \vec{v} \rangle + \langle \vec{u}_2, \vec{v} \rangle \\ &= 0 + 0 \\ \therefore \vec{u} + \vec{v} &\text{ is orthogonal to } \vec{v}. \end{aligned}$$

$$\begin{aligned} \text{if } \langle \vec{u}, \vec{v} \rangle = 0 \text{ then } \langle \alpha \vec{u}, \vec{v} \rangle &= \alpha \langle \vec{u}, \vec{v} \rangle \\ &= \alpha \cdot 0 = 0 \end{aligned}$$

$\therefore \alpha \vec{u}$ is orthogonal to $\vec{v}$.

$\therefore$ The set is a subspace.

$$\begin{aligned} 14(a) \text{ If } \langle 2\vec{u}, 3\vec{v} \rangle &= 180 \\ \therefore \langle \vec{u}, \vec{v} \rangle &= 30 \leq \|\vec{u}\|_2 \|\vec{v}\|_2 = 6.5 \\ \therefore \vec{u} \text{ and } \vec{v} &\text{ must be parallel.} \end{aligned}$$

- 13.
1. All of $\mathbb{R}^2$
 2. All lines passing through the origin
 3. $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$.

14 b) The third vector is the sum of the first two vectors and the first two vectors are orthogonal.

$$\therefore \left\| \begin{pmatrix} 5 \\ 3 \\ -4 \end{pmatrix} \right\|_2^2 = 12 + 38 = 50$$

$$\therefore \left\| \begin{pmatrix} 5 \\ 3 \\ -4 \end{pmatrix} \right\|_2 = \sqrt{50} = 5\sqrt{2}$$