

1. The distributive property

$$2. (3-j) - (2+2j) = 1-3j = \sqrt{10} \angle \arctan\left(\frac{-3}{1}\right)$$

$$3. \frac{z+1}{z - \frac{1}{1+\frac{1}{z}}} = \frac{z+1}{z - \frac{z}{z+1}} = \frac{(z+1)^2}{(z+1)z - z} = \frac{(z+1)^2}{z^2} = \frac{j^2}{-1+j} = -\frac{-1-j}{2} = \frac{1}{2} + \frac{j}{2}$$

$$4. \frac{w^* - \frac{j}{1+w^*-jz^*}}{\frac{w^*z^*}{w^*-j}}$$

5. This polynomial must have at least three real roots, so if there are complex roots, they are either one ^{complex conjugate} pair, or two complex conjugate pairs. (and two more real roots)

$$6. 2+j = \frac{1}{1-r} \quad \therefore 1-r = \frac{1}{2+j} = \frac{2}{5} - \frac{j}{5}$$

$$\therefore r = \frac{3}{5} + \frac{j}{5}$$

These are of the form

$$7. \alpha \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \quad \text{so adding two such vectors will}$$

be of the form $(\alpha_1 + \alpha_2) \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + (\beta_1 + \beta_2) \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$, and

multiplying it by a scalar is still of this form:

$$\gamma \left(\alpha \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \right) = (\alpha\gamma) \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + (\beta\gamma) \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$$

Two subspaces are two of:

$$1. \vec{0}$$

$$2. \alpha \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \text{where } \alpha \text{ is real}$$

$$3. \alpha \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \quad \text{where } \alpha \text{ is real.}$$

8. If p and q are of this form, then

$$(\alpha p)(1) = \alpha p(1) = \alpha(p(-1)) = (\alpha p)(-1)$$

$$(p+q)(1) = p(1) + q(1) = p(-1) + q(-1) = (p+q)(-1)$$

the sum of the two polynomials

that sum evaluated at 1.

these are each equal.

$\therefore$ it is a subspace.

9. $\vec{u} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$ and $\|\vec{u}\|_2 = 3$

Any vector such that no entry is greater in magnitude than 2 and the sum of the absolute values is 5.

10. $\left\langle \frac{1}{7+5j} \vec{u}, 7-5j \vec{v} \right\rangle = \left(\frac{1}{7+5j} (7-5j) \right) \langle \vec{u}, \vec{v} \rangle$
 $= \langle \vec{u}, \vec{v} \rangle$

11. The inner product of any two sequences is

$$1 + (ab) + (ab)^2 + (ab)^3 + (ab)^4 + \dots = \frac{1}{1-ab}$$

$\frac{1}{1-ab}$ cannot be zero. Thus, they are never orthogonal.

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$$\frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{v}, \vec{v} \rangle} \vec{v} \quad \text{projected onto } \vec{v} \text{ is}$$

$$\begin{aligned} & \frac{\langle \vec{v}, \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{v}, \vec{v} \rangle} \vec{v} \rangle}{\langle \vec{v}, \vec{v} \rangle} \vec{v} \\ &= \frac{\cancel{\langle \vec{v}, \vec{v} \rangle} \cdot \cancel{\langle \vec{v}, \vec{v} \rangle}}{\langle \vec{v}, \vec{v} \rangle} \vec{v} \\ &= \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{v}, \vec{v} \rangle} \vec{v} \end{aligned}$$

(The word "same" is written next to the final result with an arrow pointing to the first result.)

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$$\begin{aligned} 6 \text{ for } |\langle 2\vec{u}, 3\vec{v} \rangle| &= |2 \cdot 3 \langle \vec{u}, \vec{v} \rangle| \\ &= |2 \cdot 3| |\langle \vec{u}, \vec{v} \rangle| \\ &= 6 \end{aligned}$$

But $|\langle \vec{u}, \vec{v} \rangle| \leq \|\vec{u}\|_2 \|\vec{v}\|_2$ and they are both unit vectors.

$$\therefore |\langle 2\vec{u}, 3\vec{v} \rangle| \leq 6$$

$$14 \quad R_{3 \leftrightarrow 5} \quad \text{and} \quad R_{3 \leftrightarrow 1} \quad (\text{or } R_{1 \leftrightarrow 3})$$

↑ doesn't matter.

15 Two matrices are row equivalent if there is a sequence of row operations that can convert one matrix into the other.

$$16 \quad \begin{pmatrix} 5 & 3 & 2 & 1 \\ 2 & 1.2 & 3.3 & 5.4 \\ -4.5 & -2.7 & 0 & 41 \end{pmatrix} \sim \begin{pmatrix} 5 & 3 & 2 & 1 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 1.8 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 5 & 3 & 2 & 1 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$\therefore$ the rank is 3.