

$$1. \quad w\bar{z} = (2+2j)(1-2j) = 2 - 4j + 2j + 4 = 6 - 2j$$

p. 1/4

$$w+z = 3$$

$$\therefore \frac{w\bar{z}}{w+z} = 2 - \frac{2}{3}j$$

2. Complex addition is commutative
 $\therefore$ the order does not matter.

$$3. \quad w\bar{z}^* + w^*\bar{z} = w\bar{z}^* + (w^*\bar{z})^{**} \\ = w\bar{z}^* + (w\bar{z}^*)^* \\ = 2\operatorname{Re}(w\bar{z}^*) \\ = 2(\alpha\delta + \beta\delta)$$

$$4. \quad \text{if } \vec{u} \in \mathbb{R}^5 \text{ then } (-1)\vec{u} = \begin{pmatrix} -u_1 \\ -u_2 \\ -u_3 \\ -u_4 \\ -u_5 \end{pmatrix} \text{ and}$$

$$\text{thus } \vec{u} + (-1)\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{pmatrix} + \begin{pmatrix} -u_1 \\ -u_2 \\ -u_3 \\ -u_4 \\ -u_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\therefore -\vec{u}$ must be $(-1)\vec{u}$ as the additive inverse is unique.

5. If $u_2 = 0$ for $\vec{u}$ and $v_2 = 0$ for $\vec{v}$ then

$$\vec{u} + \vec{v} = \begin{pmatrix} u_1 + v_1 \\ 0 \\ u_3 + v_3 \\ u_4 + v_4 \end{pmatrix} \text{ and this vector has a second entry equal to zero.}$$

Similarly $\alpha\vec{u} = \begin{pmatrix} \alpha u_1 \\ 0 \\ \alpha u_3 \\ \alpha u_4 \end{pmatrix}$ and this too has a second entry equal to zero. $\therefore$ subspace

However, $\vec{u} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ ~~add~~ have zero entries
 but $\vec{u} + \vec{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}$ has no zero entries

$\therefore$ not a subspace.

6. No — the zero polynomial does not satisfy $0(1) = 2$,
zero polynomial.

p. 2 of 4

7 $\|\vec{u}\|_1 = 16$

$$\|\vec{u}\|_2 = \sqrt{4+25+1+9+25} = \sqrt{64} = 8$$

$$\|\vec{u}\|_\infty = 5$$

8 $(-j)(2-j) + (1+j)(3+j)$

$$= -2 - 2j + 3 + j + 3j + 1$$

$$= 2j$$

Remember to take the conjugate of the first vector

9. $\|\vec{u}\|_2 = \sqrt{\langle \vec{u}, \vec{u} \rangle}$

or

$$\|\vec{u}\|_2^2 = \langle \vec{u}, \vec{u} \rangle$$

10. $\left(\begin{array}{ccc|c} 2 & 1 & -1 \\ 4 & 3 & -4 \\ -6 & -3 & 3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 2 & 1 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array} \right) \therefore y = -2$
 $\therefore x = \frac{1}{2}$

$$\therefore \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 4 \\ -6 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \\ +3 \end{pmatrix} \checkmark$$

11. $\langle \vec{v}, \vec{w}_1 \rangle = 0$ and $\langle \vec{v}, \vec{w}_2 \rangle = 0$ p144

then $\langle \vec{u}, \alpha \vec{w}_1 + \beta \vec{w}_2 \rangle = \alpha \langle \vec{u}, \vec{w}_1 \rangle + \beta \langle \vec{u}, \vec{w}_2 \rangle = 0$

and $\langle \vec{v}, \alpha \vec{w}_1 + \beta \vec{w}_2 \rangle = \alpha \langle \vec{v}, \vec{w}_1 \rangle + \beta \langle \vec{v}, \vec{w}_2 \rangle = 0$

$\therefore \alpha \vec{w}_1 + \beta \vec{w}_2$ is $\perp$ to both $\vec{u}$ and $\vec{v}$.
 $\therefore$ it is a subspace.

12 $\vec{0} = 0 \cdot \vec{v} = (1-1) \vec{v} = \vec{v} + (-1) \vec{v}$

$\therefore \vec{v} + (-1) \vec{v} = \vec{0}$

$\therefore (-1) \vec{v}$ must be the additive inverse of $\vec{v}$.

13 $a = j$ and $r = \frac{1}{3} - \frac{2}{3}j$

$$\frac{1}{1-r} = \frac{1}{\frac{2}{3} + \frac{2}{3}j} = \frac{3}{2} \left(\frac{1}{1+j} \right) = \frac{3}{2} \left(\frac{1-j}{2} \right) = \frac{3}{4} (1-j) = \frac{3}{4} - \frac{3}{4}j$$

$\therefore \sum_{k=0}^{\infty} j \left(\frac{1}{3} - \frac{2}{3}j \right)^k = \frac{3}{4} + \frac{3}{4}j$

14. $(3+2j)(1+j) + (1-j)(2-j)$

$$= 3 - 2 + 3j + 2j + 2 - 1 - j - 2j$$

$$= 2 + 2j$$

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$$\vec{V}_1 \leftarrow \begin{pmatrix} 3/5 \\ 0 \\ 4/5 \\ 0 \end{pmatrix} \quad \vec{V}_2 \leftarrow \begin{pmatrix} 2 \\ -1 \\ 3 \\ -1 \end{pmatrix} - \left(\frac{6}{5} + \frac{12}{5} \right) \begin{pmatrix} 3/5 \\ 0 \\ 4/5 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \\ 3 \\ -1 \end{pmatrix} - \frac{18}{5} \begin{pmatrix} 3/5 \\ 0 \\ 4/5 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 12 \\ 0 \\ 14 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ -1 \\ -11 \\ -1 \end{pmatrix}$$

... not nice.

∴ not on exam...

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$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 8.0 \\ -2 & -1 & 2 & -6.5 \\ 3 & -1 & -2 & 3.0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & 8.0 \\ 0 & 3 & -4 & 9.5 \\ 0 & -5 & 7 & -21 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ 0 & 3 & -4 & 9.5 \\ 0 & 0 & -\frac{7}{3} & \frac{7}{6} \end{array} \right)$$

$$\therefore x_3 = -0.5$$

$$\therefore x_2 = 2.5$$

$$\therefore x_1 = 1.5$$

$$\therefore \begin{pmatrix} 8 \\ -6.5 \\ 3 \end{pmatrix} = 1.5 \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + 2.5 \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + 0.5 \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$$