

1. With alternating current, every circuit of inductors, capacitors and resistors with a single input and a single output can be represented by a complex number. If two such circuits are connected in series, the complex number representing the effect of both circuits equals the sum of the two complex numbers; while if the two circuits are connected in parallel, the complex number $\frac{1}{\frac{1}{w} + \frac{1}{z}} = \frac{1}{\frac{w+z}{wz}} = \frac{wz}{w+z}$ represents the combined effect. If one circuit is

represented by $2 + 2j$, and the other is represented by $1 - 2j$, what complex numbers represents the circuits in series and what number represents the circuits in parallel. Think of the complex number as a complex version of resistance usually called *impedance*.

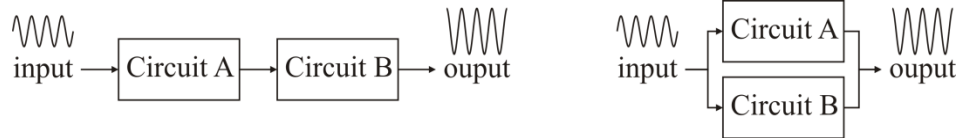


Figure 1. Two circuits in series (left) and two circuits in parallel (right).

2. Based on your knowledge of complex numbers, does it matter which order we place the circuits in Question 1 when we are placing the circuits in series? Explain your answer.



Figure 2. The output of Circuit A fed into Circuit B, and the output of Circuit B fed into Circuit A.

3. If $w = \alpha + j\beta$ and $z = \gamma + j\delta$, simplify the expression $wz^* + zw^*$ as much as possible. You may use whatever properties of the complex conjugate you wish to simplify your steps.
4. In the vector space $\mathbf{R}^5$, explicitly demonstrate the property that the additive inverse of $\mathbf{u}$ is -1 multiplied by $\mathbf{u}$; that is, $-\mathbf{u} = (-1)\mathbf{u}$. Recall that additive inverse of $\mathbf{u}$ is that vector $-\mathbf{u}$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$.
5. Which of the following, if any, are a subspace of $\mathbf{R}^4$? Justify your answer.
- The collection of all vectors $\mathbf{u}$ such that $u_2 = 0$, and
 - the collection of all vectors $\mathbf{u}$ such that there is at least one entry that is zero.
6. Is the collection of all polynomials such that $p(1) = 2$ a subspace of the vector space of all polynomials?

7. What are the 1-, 2- and infinity-norms of the vector $\mathbf{u} = \begin{pmatrix} -2 \\ 5 \\ 1 \\ 3 \\ -5 \end{pmatrix}$?

8. What is the inner product of the vectors $\mathbf{u} = \begin{pmatrix} j \\ 1-j \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} 2-2j \\ 3+j \end{pmatrix}$?

9. What is the relationship between the inner product and the 2-norm for finite-dimensional vectors?

10. Can the vector $\begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}$ be written as a linear combination of the two vectors $\begin{pmatrix} 2 \\ 4 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$?

11. Given a vector space V and given two non-zero vectors $\mathbf{u}$ and $\mathbf{v}$ in V , consider the collection S of all vectors that are perpendicular to both of those vectors. Does this set S form a subspace of the vector space V . Is this subspace equal to the entire vector space V ? Why or why not?

12. Using only the properties of vector spaces, prove that the additive inverse of a vector $-\mathbf{v}$ must be $(-1)\mathbf{v}$.

13. Recalling the geometric series, $\sum_{k=0}^{\infty} ar^k = a \frac{1}{1-r}$, what do we get if we add $j + j\left(\frac{1}{3} - \frac{2}{3}j\right) + j\left(\frac{1}{3} - \frac{2}{3}j\right)^2 + \cdots$, writing your answer in rectangular form.

14. What is the inner product of $\mathbf{u} = \begin{pmatrix} 3-2j \\ 1+j \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1+j \\ 2-j \end{pmatrix}$; that is, what is $\langle \mathbf{u}, \mathbf{v} \rangle$?

15. Apply Gram-Schmidt to the set of three vectors $\begin{pmatrix} 3 \\ 0 \\ 4 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -3 \\ 1 \\ -3 \end{pmatrix}$.

16. Convert the following problem of finding a linear combination of given vectors that equals a given right-hand vector

$$\alpha_1 \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \alpha_3 \begin{pmatrix} -3 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 8.0 \\ -6.5 \\ 3.0 \end{pmatrix}$$

into a system of linear equations in x , y and z and, separately, into an appropriate augmented matrix. Apply row operators to the augmented matrix to bring the augmented matrix into one that is in row-echelon form. You may solve this problem by either, at this point, applying backward substitution or additional row operations.

Notes:

If $\mathbf{u} \in \mathbf{R}^n$, $\|\mathbf{u}\|_1 = \sum_{k=1}^n |u_k|$ and $\|\mathbf{u}\|_{\infty} = \max_{k=1, \dots, n} \{|u_k|\}$.

$$\text{perp}_{\mathbf{u}}(\mathbf{v}) = \mathbf{u} - \text{proj}_{\mathbf{u}}(\mathbf{v})$$

$$1 \cdot \mathbf{u} = \mathbf{u}, \quad \mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}, \quad \mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}, \quad \alpha(\mathbf{u} + \mathbf{v}) = \alpha\mathbf{u} + \alpha\mathbf{v}, \quad (\alpha + \beta)\mathbf{u} = \alpha\mathbf{u} + \beta\mathbf{u}, \quad (\alpha\beta)\mathbf{u} = \alpha(\beta\mathbf{u}), \\ \mathbf{u} + \mathbf{0}_n = \mathbf{u}, \quad \mathbf{u} + (-\mathbf{u}) = \mathbf{0}_n$$