



6.4.2 Classification of matrices





Introduction

- In this topic, we will
 - Define square and rectangular matrices
 - Define diagonal matrices, tri-diagonal matrices, and upper- and lower-triangular matrices





Square matrices

- To review, a square matrix is a matrix where the number rows equals the number of columns
 - All matrices are *rectangular* matrices
 - A square matrix is a rectangular matrix, so we may end up referring to non-square rectangular matrices

$$A = \begin{pmatrix} 8.2 & -0.7 & 1.2 & 0.5 & 0.1 \\ 1.3 & 7.4 & 1.7 & 0.9 & 0.2 \\ 0.5 & 1.8 & 6.8 & 1.2 & 0.6 \\ -0.3 & 0.7 & 1.5 & 9.3 & 1.4 \\ -0.2 & 0.1 & 0.8 & 1.1 & 10.3 \end{pmatrix}$$





Zero matrix

- Like the zero vector, the $m \times n$ matrix of all zeros is described as the $m \times n$ zero matrix
 - It is denoted with an italicized O subscripted with m, n

$$\begin{array}{c} \uparrow \\ m \\ \downarrow \end{array} O_{m,n} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$
$$\begin{array}{c} \leftarrow \text{-----} \rightarrow \\ n \end{array}$$

- A square zero matrix may be represented by O_n





Matrices depending on zeros

- A matrix may be given other names to identify them depending on where the zeros of that matrix are:
 - Diagonal matrices
 - Tri-diagonal matrices
 - Upper-triangular matrices
 - Lower-triangular matrices





Diagonal matrix

- A matrix is a *diagonal matrix* if all off-diagonal entries are zero
 - The zero matrix is a diagonal matrix

$$A = \begin{pmatrix} 8.2 & 0 & 0 & 0 & 0 \\ 0 & 7.4 & 0 & 0 & 0 \\ 0 & 0 & 6.8 & 0 & 0 \\ 0 & 0 & 0 & 9.3 & 0 \\ 0 & 0 & 0 & 0 & 10.3 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 6 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1.3 & 0 \\ 0 & -0.9 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$





Diagonal matrix

- One important square diagonal matrix is the *identity matrix*
 - All entries on the diagonal are 1, all other entries are zero
 - It is denoted I_n

$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$





Tri-diagonal matrix

- A matrix is a *tri-diagonal matrix* if the only non-zero entries, if any, are on the diagonal or the sub- or super-diagonals
 - All diagonal matrices are tri-diagonal

$$A = \begin{pmatrix} 8.2 & -0.7 & 0 & 0 & 0 \\ 1.3 & 7.4 & 1.7 & 0 & 0 \\ 0 & 1.8 & 6.8 & 1.2 & 0 \\ 0 & 0 & 1.5 & 9.3 & 1.4 \\ 0 & 0 & 0 & 1.1 & 10.3 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 7 & 0 & 0 \\ 2 & -4 & 0 & 0 \\ 0 & 3 & 6 & 9 \end{pmatrix}$$
$$C = \begin{pmatrix} 1.3 & -2.8 \\ -6.4 & -0.9 \\ 0 & 3.2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$





Upper-triangular matrix

- A matrix is a *upper-triangular matrix* if the all entries below or to the left of the diagonal are zero
 - All diagonal matrices are upper-triangular

$$A = \begin{pmatrix} 8.2 & -0.7 & 1.3 & 0.7 & -0.9 \\ 0 & 7.4 & 1.7 & 1.8 & 1.1 \\ 0 & 0 & 6.8 & 1.2 & 1.5 \\ 0 & 0 & 0 & 9.3 & 1.4 \\ 0 & 0 & 0 & 0 & 10.3 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 7 & 8 & 5 \\ 0 & -4 & 9 & -3 \\ 0 & 0 & 6 & -2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1.3 & -2.8 \\ -6.4 & -0.9 \\ 0 & 3.2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$





Lower-triangular matrix

- A matrix is a *lower-triangular matrix* if the all entries above or to the right of the diagonal are zero
 - All diagonal matrices are lower-triangular

$$A = \begin{pmatrix} 8.2 & 0 & 0 & 0 & 0 \\ -0.7 & 7.4 & 0 & 0 & 0 \\ 1.3 & 0 & 6.8 & 0 & 0 \\ 0.7 & 1.7 & 1.2 & 9.3 & 0 \\ -0.9 & 1.1 & 1.5 & 1.4 & 10.3 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 5 & -4 & 0 & 0 \\ -7 & 2 & 6 & 0 \end{pmatrix}$$
$$C = \begin{pmatrix} 1.3 & 0 \\ -6.4 & -0.9 \\ 7.8 & 3.2 \\ 0.5 & 4.7 \\ 9.6 & 2.1 \end{pmatrix}$$





Summary

- Following this topic, you now
 - Know about the zero and identity matrices
 - Understand when a matrix may be described as being a
 - Diagonal matrix
 - Tri-diagonal matrix
 - Upper-triangular matrix
 - Lower-triangular matrix





References

- [1] [https://en.wikipedia.org/wiki/Matrix_\(mathematics\)](https://en.wikipedia.org/wiki/Matrix_(mathematics))





Acknowledgments

None so far.





Colophon

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